after which the above factors cause the meniscus to descend again. However, the rapid evaporation means that the thickness and length of the wetting film decrease over time [5], so the oscillations are damped and the meniscus halts at the position corresponding to the step in the given capillary.

Pressures less than those characterized by $Kn > 0.1$ do not alter the menuscus position, since in molecular flow the gas pressure difference in the capillary becomes quite small.

These measurements on capillary rise at reduced pressures make it necessary to consider the evaporation from the meniscus but show that there is a meniscus step, for which a physical explanation exists.

NOTATION

r and L, capillary radius and length; Δh, distance from meniscus to mouth of capillary; $\Delta\ell$, meniscus step; ℓ, meniscus rise; ℓ_∞ and x, maximal and relative rise; θ, wetting angle; P, gas pressure outside capillary; P_e, vapor—air mixture pressure above meniscus; ΔP, pressure difference in vapor—air mixture along capillary; P_s, saturation vapor pressure; T_ℓ, liquid temperature at meniscus; ρ gas density outside capillary; ρ_v, ρ_a, and ρ_e, densities of vapor, air, and vapor—air mixture; v, evaporation rate; Λ, molecular mean free path; Kn, Knudsen number.

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RELATIVE MOTION OF DROPS UNDER THE ACTION OF VARIABLE FORCES

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The boundary between "moderate" and "large" drops, which is fixed for each drop—medium system in gravitational conditions, shifts toward smaller drops with increase in the forces applied to the drop.

The relative motion of particles, including drops, in a medium is usually investigated in gravitational conditions, characterized by constancy of the forces applied to the particle over both time and space. The results of such investigations also from the basis for calculations in those cases where, according to the operating conditions of the apparatus, the forces acting on the particle differ from gravitational forces. The so-called standard drag curve is widely used; it consists of a dependence of the drag coefficient on the Reynolds variation over a wide range of variation of the latter. This curve is suitable for solid spheres, regardless of the nature of the applied forces and the physical properties of the medium. The situation is different for particles with a mobile interface with the medium. For bubbles and drops, the existence of a Reynolds number Re_b at which the drag coefficient begins to increase significantly with increase in Re has been established. The number Re_b, called the boundary or transition value, is assumed to be constant for each

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TABLE 1. Dependences Proposed for Determining the Boundary Reynolds Number and Drag Coefficient after Transformation Using the Dimensionless Complex A

Source	Re_r	ψ
[1]	$4{,}55\ A^{0,21}$	$0{,}29\ Re^{1,4}\ A^{-0,4}$
[2]	$2{,}92\ A^{0,238}$	$0{,}0032\ Re^{1,56}\ A^{-0,27}$
[3]	$16{,}3\ A^{0,15}$	$0{,}0568\ Re^{0,97}\ A^{-0,17}$
[4]	$11{,}7\ A^{0,161}$	$0{,}073\ Re^{1,24}\ A^{-0,282}$
[5]	$19{,}8\ A^{0,149}$	$0{,}0052\ Re^{1,4}\ A^{-0,23}$
[6]	$4{,}107\ A^{0,216}$	$0{,}0302\ Re^{3}A^{-0,715}$
[7]	$9{,}0\ A^{0,173}$	—
[8]	$4{,}04\ A^{0,214}$	$0{,}0108\ Re^{4}A^{-1}$
[9]	$7{,}35\ A^{0,232}$	—
[10]	$20\ A^{0,15}$	—

drop—medium system [1-10], and different authors have used different methods for its determination. After simple substitutions, they may be reduced to a common form in terms of the number A, which is usually regarded as a constant of the system (Table 1). The great discrepancy in the coefficients and power indices of different authors is noteworthy; it may be explained, first, by the limitation of the experimental material — each drop—medium system gives only one value of the boundary Reynolds number; and, second, in that there is actually some range of Re within which change in conditions occurs but, because of the difficulty of sharp definition of this range, the arbitrary value Re_b is used. Analogously, expressions are obtained for the frontal-drag coefficients ψ at Reynolds numbers larger than the boundary value, and they are reduced to dependences on the Reynolds number and A (Table 1). Divergence in the constants is again seen; this is due to instability in the behavior of the drops and the small range in variation of Re.

The dependences in Table 1 may be written in the general form

$$Re_b = mA^{\alpha}, \tag{1}$$

$$\psi = n\, Re^{\beta} A^{-\gamma}, \tag{2}$$

where m, n, α, β, γ are constants; $A = \sigma^3\rho^2/(\mu^4\Delta\rho g)$.

The form of the results in Eqs. (1) and (2) is convenient for the representation of the behavior of Re_b and ψ under the assumption that these dependences reflect the force interaction between the drops and the medium not only for the gravitational field. Replacing acceleration due to gravity by the acceleration j, corresponding to the force actually acting on the drop, the following conclusions may be reached: a) with increase in the forces applied to the drop, the boundary Reynolds number becomes a variable, decreasing with increase in the acceleration j; b) the drag coefficient increases here. In the simplest case, this means that, if a constant force exceeding the gravitational force acts on the drop, the boundary Reynolds number becomes less than in purely gravitational conditions, and the drag coefficient increases. For comparison, the drag coefficient for a solid sphere decreases in these conditions, or remains constant (at large Re).

In applied problems associated with the introduction of drops in a flow or effervescence in a motionless medium, the force interaction between the drops and the medium is complicated by inertial effects, which affect the value of the drag coefficient. Nevertheless, there is a possibility of modeling the conditions in which forces of variable magnitude are equalized practically instantaneously by the drag: to this end, the drop must be introduced in a rotating liquid [11]. The centrifugal force acting on the drop is proportional to the centripetal acceleration ω^2R, which determines the boundary Reynolds number and the drag coefficient

$$Re_b \sim (\omega^2 R)^{-\alpha}, \tag{3}$$

$$\psi \sim Re^{\beta}(\omega^2 R)^{-\gamma}. \tag{4}$$

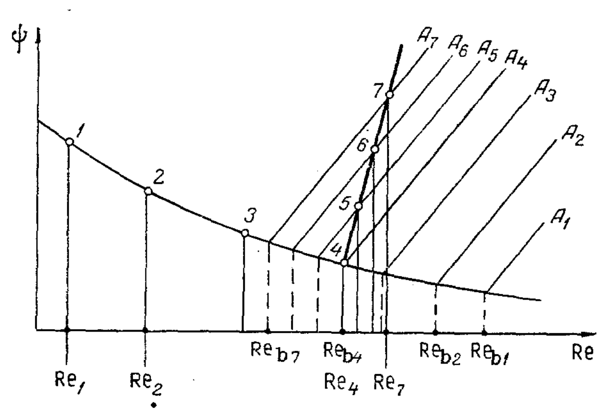

Fig. 1. Diagram of the change in conditions of drop deposition in a centrifugal field: 1-7) successive positions of the drop and corresponding values of Re, Re_b, and A.

Two variants of drop motion are possible: heavier drops move from the center of the rotor to its periphery and lighter drops in the opposite direction. In both cases, using Eqs. (3) and (4), Re_b and ψ may be determined for any R if the numerical values of the coefficients and power indices are known. A diagram of the variation in Re and the attainment of Re_b is shown in Fig. 1. Suppose that initially the drop position is determined by the radius R_1, which corresponds to the Reynolds number Re_1 and point 1 on the drag curve, as well as boundary Reynolds number Re_{b1}. Moving away from the center of rotation, the drop reaches radius R_2, which corresponds to Reynolds number Re_2 and point 2 on the drag curve. According to Eq. (3), Re_b decreases because $R_2 > R_1$, and takes the position denoted by Re_{b2}. As the drop moves away from the center of rotation, Re and Re_b move closer together, and at some point 4 they coincide. This means that boundary conditions have been reached, and then the dependence of the drag coefficient ψ on the Reynolds number must take the form in Eq. (4). With further increase in distance to the axis of rotation of the medium, the boundary Reynolds number continues to decrease (Re_{b6}, Re_{b7}), and the actual Reynolds number increases (Re_6, Re_7). However, in this case too, the influence of the boundary Reynolds number is felt: the drop motion is determined by the different position of the curves A_6, A_7,... . Hence it follows that increase in drag coefficient in the centrifugal field in the section $Re > Re_b$ is more significant than in the gravitational field. In Fig. 1, this is clearly demonstrated by the difference in slope of the segments 4-5-6-7 and A = const. The sharp increase in drag over the radius indicates slight change in drop velocity with relative motion in the given conditions. This is confirmed by experiment [12].

The motion of the lighter drop occurs in the opposite order, so that Fig. 1 may again be regarded as an illustration in this case. Note that the drop reaches the initial position 7 after a small acceleration section.

The centrifugal field gives the following advantages: a) it is possible to have a set of values of Re_b in a single drop—medium system (in a gravitational field, there is only one); b) it is possible to avoid extrapolation of the results obtained in a gravitational field and to perform the investigation in a wide range of forces acting on the drop.

In conditions of variable forces, it is expedient to reexamine the question of the use of A, which is justified in a gravitational field in that it is constant for each drop—medium system. In conditions of variable force, this is no longer true of A but is true instead of the Laplace number, containing the drop dimension d, which remains constant in the course of drop motion. The Laplace number Lp and A are related, so that the results may always be converted

$$\mathrm{Lp} = \frac{\sigma d \rho}{\mu^2} = \sqrt[3]{\frac{6}{\pi} A \psi \,\mathrm{Re}^2}. \tag{5}$$

The segment 4-5-6-7 (Fig. 1) is characterized precisely by constant Laplace number.

The general form of the dependence for the drag coefficient in terms of the Laplace number at above-boundary Reynolds numbers is obtained from Eqs. (2) and (5).

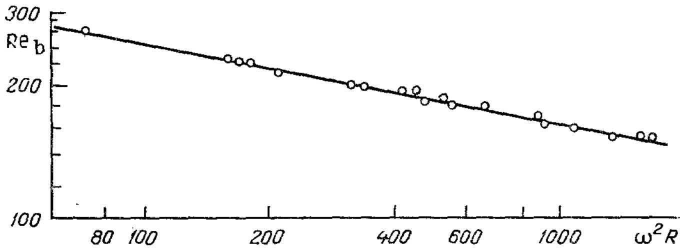

Fig. 2. Dependence of the boundary Reynolds number on the centripetal acceleration. In gravitational conditions, $Re_b \approx 500$; water drop in ketosine. ω^2R, m/sec^2.

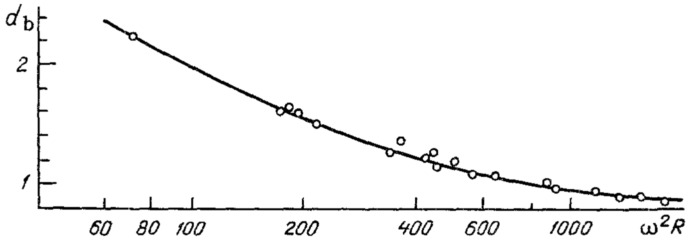

Fig. 3. Dependence of the boundary diameter on the centripetal acceleration. In a gravitational field, $d_b \approx 4.2$ mm; water drops in kerosine, d_b, mm.

$$\psi = n\,\mathrm{Re}^{\beta}\mathrm{Lp}^{-\gamma}. \tag{6}$$

The boundary Reynolds number is determined by the general value of ψ from Eq. (6) and the drag curve. Assuming that

$$\psi = a\mathrm{Re}^{-\eta}, \tag{7}$$

it follows that

$$\mathrm{Re_b} = m\,\mathrm{Lp}^{\alpha}. \tag{8}$$

After modifying the known dependences for centrifugal-field conditions, it is found that the constants in Eqs. (6) and (8) differ in the same way as the constants for ψ and Re_b in the formulas in Table 1. In addition, the modification in this case is associated with extrapolation of the dependences obtained for a narrow range of variation of the basic parameters. Therefore, the results of measuring the dynamic characteristics of a drop freely settling in a uniformly rotating liquid may be used to determine the constants in Eqs. (6) and (8). The corresponding experimental method was outlined in [13, 14].

The experimental data confirm the fundamental conclusion that Re_b increases with increase in the force acting on the drop. As an example, results are shown in Fig. 2 for the case when the centrifugal force is the only variable quantity. Overall, the volume of experimental material permits a reduction in Re_b from 500 to 40, in Lp from 75,000 to 400, and in A from 10^{10} to 10^4. The following theoretical dependences are obtained here

$$\mathrm{Re_b} = 3{,}6\,\mathrm{Lp}^{0{,}4} = 10A^{0{,}16}. \tag{9}$$

For $Re > Re_b$

$$\psi = 0{,}11\,\mathrm{Re}^{2{,}8}\mathrm{Lp}^{-1{,}4} = 0{,}18\,\mathrm{Re}^{1{,}27}A^{-0{,}32}. \tag{10}$$

As usual, the unique value determined by Eqs. (2) and (7) is taken as the boundary Reynolds number. In fact, the transition from one set of conditions to another is not so sharp, and occurs in some range of Re. This complicates the determination of Re_b, and therefore the maximum spread in the values of up to 20% given by Eq. (9) must be regarded as perfectly satisfactory.

Using Eqs. (9) and (10), the equation of drop motion under the action of variable forces may be solved, taking account of the change in deposition conditions.

Some interesting consequences follow directly from Eq. (9). After expansion of the generalized variables, a relation is established between the drop diameter and the centri-

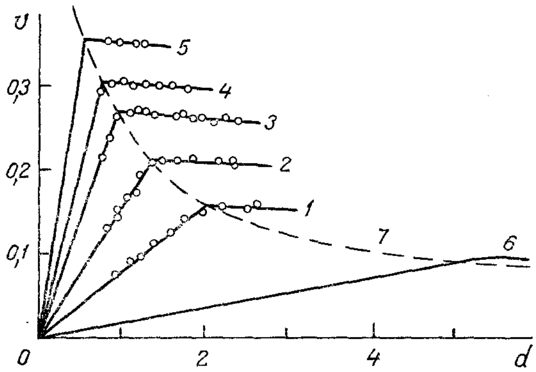

Fig. 4. Dependence of the drop velocity on its diameter at various centripetal accelerations $\omega^2 R$, m^2/sec: 1) 100; 2) 200; 3) 500; 4) 1000; 5) 2000; water drop in kerosine. v, m/sec; d, mm.

petal acceleration in conditions of Re_b

$$d_b = a(\omega^2 R)^{-0,4}, \tag{11}$$

where a is a constant consisting of the physical properties of the drop–medium system. In contrast to a gravitational field, the boundary diameter is a variable, decreasing with increase in the forces acting on the drop (in this case, centrifugal). In Fig. 3, Eq. (11) is shown for a specific system. With moderate rotation of the rotor, the boundary diameter is reduced by practically an order of magnitude. Such concepts as "moderate" and "large" drops thereby lose the geometric meaning which they have in a gravitational field. Drops less than a millimeter in size are "large" in a centrifugal field; these form most of the drops in some types of centrifugal extractors [15].

The velocity of drops of diameter d_b has the following peculiarity. It is known that, in a gravitational field, each drop corresponds to a single velocity value, while the drop of diameter d_b is the fastest. In a centrifugal field, there is the possibility of change in drop velocity with fixed diameter, i.e., any drop may correspond to the boundary velocity. The relation between the velocity v_b, the diameter d_b, and the centripetal acceleration at Re_b follows from Eqs. (9) and (11)

$$v_b = b d_b^{-0,6}, \tag{12}$$

where b is a constant consisting of the physical properties of a specific drop–medium system. Equation (12) for the given system is analogous to the v_b–d_b relation in a gravitational field (Fig. 4). Curve 7, passing through the maximum of the v–d curves for discrete values of the centripetal acceleration, corresponds to Eq. (12). For comparison, curve 6 corresponds to a gravitational field.

Decrease in boundary value of the Reynolds number leads to the consequence that in conditions of motion so-called "large" drops are those usually regarded as "moderate." Correct characterization of the drop-deposition conditions facilitates more accurate calculation of the productivity and efficiency of mass-transfer equipment in which an energy supply is used to increase the relative velocity of the phases.

NOTATION

ρ, μ, density and viscosity of the medium; σ, interphase tension, N/m; $\Delta\rho$, difference in density of drop and medium, kg/m^3; v, drop velocity relative to medium, m/sec; d, diameter of sphere of the same volume as the drop, m; j, acceleration due to the force applied to the drop, m/sec^2 – in particular, the acceleration due to gravity g or the centripetal acceleration $\omega^2 R$; ω, frequency of rotation of rotor liquid, sec^{-1}; R, distance from drop to axis of rotation, m; ψ, frontal drag coefficient of drop; Re, Reynolds number; Lp, Laplace number. Subscripts: b, boundary between conditions of "moderate" and "large" drops.

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USING THE ISENTROPIC INDEX TO CALCULATE THE PARAMETERS OF TWO-PHASE FLOW

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A method of determining the volume vapor content in two-phase mixture is proposed, on the basis of the relation between the isentropic index of the mixture and the "frozen" sound velocity there.

In the gas dynamics of an ideal gas, the isentropic index

$$k = -\frac{v}{p}\left(\frac{\partial p}{\partial v}\right)_s \tag{1}$$

is used as an effective quantity permitting sufficiently fast and reliable calculation, using gas-dynamic functions, of all the necessary parameters of ideal-gas flow: the pressure, temperature, and density [1, 2]. Here k is usually assumed to be constant for each specific gas over the whole of the practical pressure and temperature ranges, and depends basically only on the number of rotational degrees of freedom of the gas molecule [2]. For water vapor ($\delta_{ro} = 3$)

$$k = \frac{5+\delta_{ro}}{3+\delta_{ro}} = \frac{8}{6} = 1{,}333\ \ldots$$

Taking into account that the gas (vapor) system is only a particular, limiting case of the multiphase liquid—gas (vapor)—solid particle system, it is natural that the scope of use of the isentropic (adiabatic) index of the mixture must be broadened for such systems. In this case, it becomes a function of the temperature, pressure, and gas content [2, 3].

Consider the propagation of a weak perturbation wave in a two-phase mixture, assuming that the distribution of one phase in the other is arbitrary and the mixture may be both in a state of rest and in a state of motion.

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